Online Appendix to "Index-Based Price Adjustment in US Strategic Petroleum Reserve Drawdowns"

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A Proofs

A.1 Proof of Proposition 1

I first show that expected information rents must be smaller in the basis auction. This follows from a general property of the order statistics of independent random samples from log-concave distributions. The expected information rent is equal to the expected difference between the first and second order statistics of bidders' types. The conclusion follows from Assumption 1 and the following lemma.

Lemma 1. *Suppose that $Y_1, \ldots, Y_N$ and $X_1, \ldots, X_N$ are samples of independent random variables, with $Y_i = X_i + U_i$ for $U_i \perp X_i$ and X_i having a log-concave density. Then:*

$$E\left[Y_{(n:n)} - Y_{(n-1:n)}\right] \geq E\left[X_{(n:n)} - X_{(n-1:n)}\right]$$

where $Y_{(n:n)}$ and $Y_{(n-1:n)}$ are the nth and $(n-1)$th order statistics.

Proof. By Theorem 3.B.7 of Shaked and Shanthikumar (2007), Y dominates X in the dispersive order: $X \leq_{disp} Y \equiv X + U$. Define $U_{i:n} := X_{i:n} - X_{i-1:n}$ and $V_{i:n} := Y_{i;n} - Y_{i-1:n}$. By Theorem 3.B.31, $V_{i:n}$ dominates $U_{i:n}$ in the stochastic order for $i \geq 2$. □

Next, I show that the expected interim valuation of a winning bidder is smaller in the fixed price auction. This is because:

$$\begin{aligned} E\left[S_{(n:n)} + \left(\frac{N-1}{N}\right) E\left[\omega_j | s_j \leq S_{(n:n)}\right]\right] &= E\left[\frac{1}{N} E\left[\omega_i | s_i = S_{(n:n)}\right] + \left(\frac{N-1}{N}\right) E\left[\omega_j | s_j \leq S_{(n:n)}\right]\right] \\ &\quad + E\left[E\left[u_i | s_i = S_{(n:n)}\right]\right] \\ &= E\left[\omega_i\right] + E\left[E\left[u_i | s_i = S_{(n:n)}\right]\right] \\ &\leq E\left[\omega_i\right] + E\left[U_{(n:n)}\right] \end{aligned}$$

where the second equality follows from the law of total probability.The basis price auction therefore generates greater surplus in expectation, while reducing expected information rents. Hence, expected revenue improves.

A.2 Uniform price auction model

I first characterize equilibrium of the fixed price auction before establishing the revenue comparison result. Consider a bidder who observes signals ω_i and u_i. The optimal quantity bid at clearing price p is the solution to

$$\max_{q_i} \quad \left(E\left[\bar{\omega}|\omega_i, u_i, p\right] + u_i - p\right) q_i - \frac{\lambda}{2} q_i^2$$

A necessary condition of equilibrium is then:

$$q_i = \left(\frac{1}{\lambda + \psi_i}\right) \cdot \left\{E\left[\bar{\omega}|\omega_i, u_i, p\right] + u_i - p\right\} \tag{1}$$

where $\psi_i = \frac{dp}{dq_i}$. In a symmetric equilibrium, $\psi_i = \psi_j = \psi$ for all bidders i and j. I conjecture that (1) is linear: $q_i = a + b\omega_i + cu_i - Dp$ for constants (a, b, c, D). If this strategy is adopted by all bidders, the market clearing price p is a linear function of Gaussian random variables. Moreover, $\bar{\omega}$ is Gaussian conditional on ω_i, u_i. Differentiating the market clearing condition with respect to q_i gives $\psi = \frac{1}{(N-1)\cdot D}$. By the projection theorem,

$$E\left[\bar{\omega}|\omega_i, u_i, p\right] = \omega_i + \frac{Cov\left(\bar{\omega}, p|\omega_i, u_i\right)}{Var\left(p|\omega_i, u_i\right)} \cdot \left(p - E\left[p|\omega_i, u_i\right]\right) \tag{2}$$

Define $\epsilon_i := w_1 - \omega_i$. The key observation is that $\omega_j = w_1 + \epsilon_j = \omega_i - \epsilon_i + \epsilon_j \Rightarrow \omega_j|\omega_i, u_i \sim N\left(\omega_i, 2\sigma_\omega^2\right)$ and $Cov\left(\omega_j, \omega_k|\omega_i, u_i\right) = Cov\left(\omega_i - \epsilon_i + \epsilon_j, \omega_i - \epsilon_i + \epsilon_k|\omega_i, u_i\right) = \sigma_\omega^2$. Hence:

$$\frac{1}{N}\sum_{j\neq i} \omega_j|\omega_i, u_i \sim N\left(\omega_i, \left(\frac{N-1}{N}\right)\sigma_\omega^2\right)$$

It follows that the right hand side of (2) is linear in the conditioning variables:

$$E\left[\bar{\omega}|\omega_i, u_i, p\right] = \gamma_0 + \gamma_w\omega_i + \gamma_u u_i + \gamma_p p$$

where the coefficients $(\gamma_0, \gamma_1, \gamma_u, \gamma_p)$ can be expressed in terms of (a, b, c, D) and the model primitives. Matching coefficients reveals that $\tilde{b} := b/D = 1$ while $\tilde{c} := c/D$ is the unique real root of the cubic equation

$$\tilde{c}^3 r - \tilde{c}^2 r + \tilde{c} - N = 0$$

where $r := \sigma_u^2/\sigma_\omega^2$. The remaining equations pin down $\tilde{a} := a/D$ and D conditional on $\tilde{c}$. This completes the characterization of the equilibrium bidding strategy.

It follows that the expected clearing price in the fixed price auction is

$$E[p] = E[\bar{\omega}] + \theta_u - \lambda \cdot \frac{Q}{N}\left(\frac{N-1}{N-2} + \Psi(N,r)\right) \tag{3}$$

where

$$\Psi(N,r) = \frac{4Y(N,r)^{1/3} - 2^{2/3}Y(N,r)^{2/3} + 6\sqrt[3]{2}N \cdot r - 2\sqrt[3]{2}}{10Y(N,r)^{1/3} - 2^{2/3}Y(N,r)^{2/3} + 6\sqrt[3]{2}N \cdot r - 2\sqrt[3]{2}} - \frac{N-1}{N-2}$$

for $Y(N,r) = 2 + 9N(3N-1)r - Z(N,r)$ where

$$Z(N,r) = 3\sqrt{3}N\sqrt{(4Nr^2 + (9N(3N-2)-1)r + 4)r}.$$

In comparison, the expected clearing basis in the basis auction is

$$E[d] = \theta_u - \lambda \cdot \frac{Q}{N}\left(\frac{N-1}{N-2}\right) \tag{4}$$

The difference between $E[\bar{\omega}] + E[d]$ and (3) is (5).

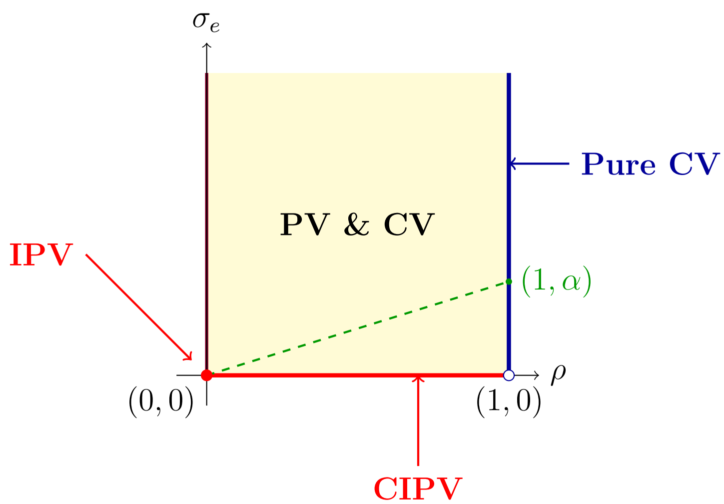
It remains to characterize the limiting behavior of $\Psi(\cdot,\cdot)$. It is not difficult to show that $\Psi(N,r)$ is strictly positive when $Y(N,r)$ is sufficiently negative. It therefore suffices to prove that $\lim_{N\to\infty} Y(N,r) \to -\infty$ with $\frac{d}{dN}[Y(N,r)] < 0$ for sufficiently large N. This must be true because when r is fixed

$$Z(N,r) = 27r \cdot N^2 + (2r-9)rN + O(1)$$

and therefore $Y(N,r) = -2r^2N + O(1)$.

Next, consider a fixed N. For large r the numerator of the first fraction of $\Psi(N,r)$ is approximated by $\sqrt[3]{3} \cdot \sqrt{2N}\left(4 - \sqrt[3]{4} \cdot (9N-3)\right) r^{1/2}$ while the denominator is approximated by $\sqrt[3]{3} \cdot \sqrt{2N}\left(10 - \sqrt[3]{4} \cdot (9N-3)\right) r^{1/2}$. The ratio equals $\frac{N-1}{N-2}$; hence $\lim_{s\to\infty} \Psi(N,s) = 0$.

Appendix Figure 1: Information Structures in the Vives (2011) Model



B Power analysis for Hickman et al. (2021) test

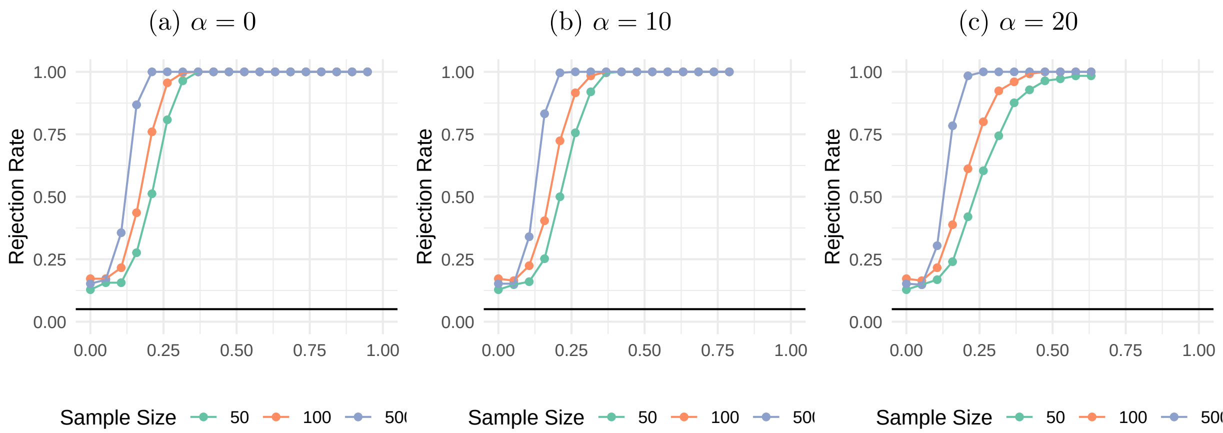
This section evaluates the power of the multiunit bid intercept test described in Section 4.2.

To simulate bid data, I adopt the uniform price auction model of Vives (2011). Bidders are assumed to have marginal valuations of the form $v_i(q) = \theta_i - \lambda q$. The intercept θ_i is given by $\theta_i = \phi + \gamma_i$ where $\phi \sim N(\mu, \rho\sigma_\theta^2)$ is a common shock affecting all bidders and $\gamma_i \sim N(0, (1-\rho)\sigma_\theta^2)$ is idiosyncratic and independent across bidders. It follows that $corr(\theta_i, \theta_j) = \rho$. Bidder i observes a signal $s_i = \theta_i + \epsilon_i$ where $\epsilon_i \sim N(0, \sigma_e^2)$ is iid noise.

Appendix Figure 1 depicts the information structure in the space of ρ and σ_ϵ, where ρ denotes the correlation $\text{cor}(\theta_i, \theta_j) = \rho$ for all $i \neq j$. The origin corresponds to independent private values. The vertical line that begins at $(0, 0)$ encompasses IPV models in which bidders do not perfectly observe their own valuations. The x-axis, where $\sigma_\epsilon = 0$, coincides the family of conditional IPV models with increasingly greater correlation between bidder values. The limit of perfect correlation in the CIPV model is a model of pure common values in which all bidders are perfectly informed about the value of the good. The vertical line that begins at the point $(1, 0)$ encompasses classical pure common values with uncertain valuations, with the severity of the winner's curse increasing in σ_ϵ. Finally, the interior shaded region encompasses a class of mixed information structures with both private and common values.

The simulation design is as follows. To evaluate the power of the model, I consider the ability of the Hickman et al. (2021) test to reject sequences of models that correspond to rays through the origin like the dashed green line in Appendix Figure 1. For instance, the ray

Appendix Figure 2: Simulated Rejection Rates for $(\rho, \sigma_e) = (t, \alpha t)$



Notes: (1) As t increases (x-axis), the information structure is further from IPV. (2) Approximate rejection rates from 250 simulations.

along the x-axis defined by $(t, \alpha t)$ for $\alpha = 0$ encompasses the family of CIPV models described above. Motivated by the data, I generate samples of S auctions for $S \in \{50, 100, 500\}$. I fix $(\lambda, \mu, \sigma_\theta) = (100, 0, 10)$. The total quantity sold in each auction is fixed at $Q = 1000$ and the number of bidders is fixed at $N = 5$. When N is small relative to the amount of private information, a symmetric linear BNE may not exist; I drop auctions for which this occurrs.

Appendix Figure 2 presents simulated power curves for a 5% test for each sample size along the rays $\alpha \in \{0, 10, 20\}$. Even for the case of $S = 50$ auctions, the test attains rejection rates above 75% for $\rho > 0.25$ in the CIPV model (i.e, $\alpha = 0$). As α increases for the same level of ρ, corresponding to the introduction of private information, rejection rates decrease slightly but remain high for modest deviations from private values. The rejection rate at $t = 0$ is around 10% for all designs, above the nominal level. In unreported results, I find that the rejection rate appears to converge to the nominal level for larger sample sizes.

References

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